A Comprehensive, Conclusive Analysis Of Ordinal Voting Methods

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1. Introduction

Although it certainly could be argued that there are better ways to make good decisions, we employ the mechanism of voting when it is important to diffuse decision-making power among large numbers of people, thereby preventing its concentration within a small faction or an individual. Clearly, the primary objective of voting must be to make the best possible decision as it affects all those who vote. We have no choice but to assume (regardless of veracity) that those voting are at least somewhat interested, informed and rational; therefore able to contribute toward making a good decision. The best possible decision must then be defined as the outcome which results in the highest satisfaction, net of dissatisfaction, summed over all those who voted.

The simplest possible case is a binary decision, a referendum. Suppose that 51% of voters vote "No" and 49% vote "Yes" on some referendum. By traditional "majority rule" the measure is defeated.

However, further suppose that the 51% who voted "No" don't really have a very strong opinion regarding this particular issue. On the other hand, the 49% who voted "Yes" believe it to be a super-good thing and favor it very strongly. In this case, the majority rule decision to defeat the referendum would not likely be the best choice by the previously stated definition.

Now suppose that a machine, call it a "satometer," is available which is trusted by voters and which can accurately measure each voter's satisfaction regarding the issue at hand on some absolute scale of "sats" (dissatisfaction would be negative "sats"). If the net total sats for all voters is positive, the referendum result is "yes" and if total sats is negative, the result is "no" (a zero-sat tie would be resolved by falling back to majority rule, subtracting the number of voters who registered negative sats from those who registered positive sats). This hypothetical referendum illustrates the ideal best possible decision that we aspire to most closely approach in real-world voting. It is based upon complete, accurate, comparable and sincere cardinal information measuring the satisfaction of each voter on the given issue.

Considering now the general case where voters are to select n of m options, we set up our voting booth satometer to grab a quick reading on each of the m options from each voter. In essence, it's a separate referendum on each option. This is indeed a large and complete data set. The winners will be the n options with the highest net sat totals (or in extremely sad cases, the positive-most or least negative totals). A bitter lesson swiftly learned is not to nominate candidates with large negatives. Buh bye Donald J. Trump and Hillary R. Clinton; evidence indicates that both almost undoubtedly would have had negative net sat totals in the 2016 election. The sun breaks forth from behind the turbulent dark clouds of polarization and the lands grow happier and more peaceful.

Lacking a satometer, obtaining such information-rich and *sincere* cardinal data from voters is extremely problematic. At least one method has been proposed (True Weight Voting) which promises to obtain and utilize valid cardinal data. Possibly, it could closely approach the ideal behavior described above. However, it is untested and enough of a "radical" change that it is unlikely to find acceptance in the near term. Therefore, the remaining discussion will focus on how best to utilize more familiar and easier-to-collect ordinal data, but will always strive to most closely approach the above ideal decision model.

Any voting method can be considered as two steps: 1) gathering data from voters, and 2) processing that data in some way to yield the result. These steps are not entirely independent. Obviously, the data collected can affect the processing and vice versa. Perhaps just slightly less obvious is the fact that voters' understanding of the processing can affect the sincerity of the data they provide. However, if it can be shown that, for a certain combination of the two steps, the best possible data is being gathered and that this data is being processed in the best possible way to yield the result, then we can claim to have defined the best possible ordinal voting method. "Fairness" is the shiny object that has by far captured the most attention; much less effort has been put into producing the best decisions as defined above.

2. Gathering Data From Voters

Cardinal data is lost when voters are restricted to providing ordinal preferences. It is abundantly obvious that, when restricted to ordinal data, there is, unfortunately, way much less of it. We have only a pale shadow of the satometer's rich and complete data from which we must infer and estimate as best we can what the missing cardinal data most likely looks like and what decision it would render. Here are some important considerations affecting this step:

- Since the data which can be obtained is so limited, it is critical that all useful ordinal data be collected and utilized
- Voters must understand what information they are to provide and, to a reasonable extent, how it will be used to determine the result
- Voters must be capable of providing valid data and be willing to provide it
- Voters must be motivated to provide sincere (and not insincere or "strategic") information

It will not be possible to obtain a complete ranking of all options from all (or even most) voters. If write-in choices are freely allowed, it is not even theoretically possible as voters have no way of knowing what or how many different write-ins other voters may have supplied, not to mention knowing enough about them to intelligently rank them. Also, many voters will only have a first choice, or possibly a first and a second choice in mind. They may not have sufficient knowledge of other options to be able to rank them on any rational or meaningful basis. It is counterproductive to force voters to provide rankings as the data will be just unhelpful noise which can't improve and likely would degrade decision quality.

Clearly, voters must be allowed to voluntarily supply their first choice as it is the most important single datum. Equally clearly, a single choice is not enough, as it would limit us to brain-dead Plurality. So, voters must also be allowed to voluntarily supply a second choice, if they have one. For "good measure," voters will be allowed to voluntarily supply a third choice, but that will be the limit. Few will likely have a third choice, but it will be valid data to be considered for those voters who are motivated enough to provide it. A vanishingly small number of voters are likely to have a fourth choice. As will be evident in the next section, the impact upon the decision that fourth and lower choices could have is miniscule, so we need not mourn their absence.

Although not by mathematical rigor, straightforward logic applied to the characteristics of real-world voters and elections supports reasonable confidence that an optimum data set to collect has been specified. The remaining task as it relates to collecting data is to make sure its processing motivates voters to supply sincere and not strategic information.

3. Processing The Data

Consider a voter's first choice. A satometer would have measured some absolute number of sats. Since that number is not available, represent this as S_1. Worse, the information is not available for any voter, so we are forced to assume that the first choices of all voters are the same, namely S_1 sats (although certainly they are different).

Similarly, we don't know the strength of the voter's support for the option ranked second, so we label that, S_2. We can calculate the value of S_2 for any given voter from the value of S_1 (for the same voter) by multiplying S_1 by some factor we will call a relative weighting factor. That is, $S_2 = wS_1$, where w is the weighting factor. What can be said about the value of w?

At one extreme, the voter may have liked the top two choices equally and may have flipped a coin to decide which option to rank first. In this case, $S_2 = S_1$ and, therefore, $w = 1$. We can be sure that w can't have a value greater than 1; if it did, the voter would have ranked the second option first.

At the opposite extreme, the voter may not have liked the second choice option much at all and may have seriously considered leaving that option out of the ranking altogether. Since, in fact, the voter's decision was to include the option, it must enjoy some positive amount of support (however small) from that voter. Therefore, w must have a value greater than zero, so $w > 0$.

Thus, $0 < w <= 1$. Again, since we have no better information, we are forced to assume that w is the same for all voters and all rankings, although we know that it surely is not. However, the situation may not be as bleak as that sounds. We can still make reasonable decisions when statistically large numbers of voters are involved if we know the average value of w for all voters. Statistically, it is highly unlikely that the average w would either be close to 1 or close to 0. In fact, the best assumption that can be made is that the average value of w is 0.5. That is, averaged over lots of voters for many races in many elections, voters' support for the options they rank second is half as strong as their support for the option they rank first.

By repeating exactly the logic we have just been through, we can conclude that the weight of a third choice should be assumed to be half of the weight of a second choice (and, therefore, a quarter of the weight of a first choice). In fact, this can be extended to as many choices as one would like (i.e., a fourth choice is half the weight of a third choice or an eighth of a first choice, etc.) Even if valid data could be obtained for such lower choices, their influence on the decision declines exponentially so we need not be concerned by the absence of this data.

The satisfaction (sats) each voter had for each option were lost forever in the voting booth when the voter boiled them down to just an ordinal ranking. There is no way to reconstruct that or the total (cardinal) satisfaction of all voters for each option. However, by using proper weightings for the choices, a proxy for the total of all voters' cardinal satisfaction data can be constructed for each option. The proxy value for an option is meaningless by itself, but the *relative* proxy values for the options provides the most accurate possible way to compare the total voter satisfaction voters have for each option relative to the other options.

Most people find it easier to deal with integers rather than decimal fractions, so 4 "points" are awarded for a first choice, 2 "points" for each second choice and 1 "point" for each third choice, which accomplishes the proper weightings. The option receiving the highest point total (proxy value) should be the choice that results in the greatest voter satisfaction.

The proxy values could very well be used to directly identify the winning option. To do so would have the advantage of making the method precinct summable, and would indeed be a strong voting method. However, it is much better not do this. Instead, the proxy values are used to reliably identify the weakest option, which then is eliminated in an iterative, last-man-standing manner (similar to Baldwin or IRV – Instant Runoff Voting). This will usually, but not always, lead to exactly the same final result. The method was proposed in 2007 and is called MRCV (Minet Ranked-Choice Voting). See Appendix A for illustrative examples.

There are several reasons for the iterative eliminations approach:

- It is very important for all voters to clearly understand that, if their first choice is eliminated, their second choice is "promoted," and will then count in every way as though it had been their first choice. It is this which exorcises the irresistible pressure to vote insincerely for the "lesser-of-two-evils."
- It would be a hard sell to convince people to abandon majority rule, so the iterative system eliminates options until one has a majority of the first choices (or, more rarely, is the only one remaining). In fact, the scoring system is not even used if an option initially has an outright majority of first choices. Thus, it is easy for voters to see and feel comfortable that MRCV is built upon and supports majority rule.

- In close races where two options have nearly equal high points, it is possible that the wrong winner could be chosen (although the only thing that could reveal such an error would be the missing original cardinal data). It is safer to instead use the scoring system to identify the weakest option. An error in eliminating the worst option is not a problem, as long as the error is not so egregious as to eliminate the option that should win (a huge problem with IRV). MRCV normally will still converge on the same correct winner. As weak options are eliminated and choices are promoted, the correct winner is ultimately identified.
- Meticulously eliminating the weakest option one at a time makes MRCV as resistant as possible to irrelevant alternatives (IIA). It also optimizes handling of clone situations.

4. Concise Definition of MRCV Procedure

Each elector may rank his or her first, second and third choices. Any choice may be a write-in. A choice may not be repeated. Electors are not forced to rank three choices, but are limited to a maximum of three. The outcome will be determined by the following (sometimes) iterative procedure:

1. **Determine Winner** – Total the first choices. If any option has a majority of the first choices, it is the winner. If only one option remains, it is the winner. If there is no winner, proceed to step 2.
2. **Eliminate Weakest Option** – Assign weighting points to the choices on each ballot. A first choice receives 4 points, a second choice receives 2 points and a third choice receives 1 point. Total the points for each option across all ballots. Eliminate the option having the lowest point total. If there is a tie, eliminate the tied option having the smallest total of first choice votes. If still a tie, eliminate the tied option that has the smallest total of second choice votes. If a tie persists, eliminate one of the tied options at random.
3. **Promote Choices** – If a ballot's first choice has been eliminated, promote the second choice (if any) to first. For any ballot where the second choice has been promoted or eliminated, promote the third choice (if any) to second. Any eliminated or promoted choice that has no lower choices will cease to exist. Return to step 1.

5. The Jones Rule

The (Douglas) Jones rule states that anything having to do with elections must be understandable by a bright high school student. At least a good majority of voters need to be able to understand the voting system in order to have full confidence in it. Judges of elections, poll watchers and all those responsible for running, supervising or auditing elections certainly need to understand it. And of course, legislators should not enact anything they can't understand.

Simplicity likely is the main reason Plurality is still widely used. Unfortunately, some increase in complexity is absolutely necessary to achieve improved results. Fortunately, the above MRCV procedure (just the procedure, not the deeper explanation behind it) is simple enough that it can be well understood by most when explained. It probably squeaks in under the Jones rule.

6. Voter *Dis*satisfaction

With a satometer, it was possible to accurately measure both satisfaction and dissatisfaction on the same scale. Use of both enabled greatly improved decisions, “encouraged” the nomination of more widely acceptable candidates and promised to, over time, generally reduce polarization. (Anyone for True Weight Voting?) One might wonder why, while collecting ordinal data, MRCV does not offer voters some opportunity to register dissatisfaction. Perhaps they could optionally provide a separate ranking of any candidate(s) they would *not* want to win. While it may be possible to logically implement such a feature, it is probably not a good idea. Although there may be other reasons as well, the overriding reason is to hold the increase in complexity to a bare minimum so as to maintain compliance with the Jones rule.

7. Implementation

MRCV could be done using rustic means such as hand-marked paper ballots. However, use of modern technology is strongly recommended, as long as it is carefully done (discussed extensively elsewhere). A good implementation can handle precinct non-summability smoothly and confer other advantages as well. Very briefly, the flow is:

1. Voters vote electronically. The equipment produces a voter-verifiable paper ballot.
2. Each voter reviews and verifies the paper ballot, then drops it into a traditional ballot box.
3. When the polls close, a check-summed XML file of all ballots (randomized) is produced.
4. The XML file is printed and posted at each polling place, and is also made publicly available.
5. The XML files from all polling places are processed at a central location to yield official results.
6. The paper ballots are used to verify each polling place’s XML file. The central processing can be verified by anybody anywhere.

A comprehensive software implementation of MRCV is available called Election Manager. EM is capable of running entire elections. Reliable final results are available minutes after the last poll closes. All results can be verified (completely or statistically). EM may be used royalty-free for non-commercial purposes.

8. Other Interesting Observations

The value chosen for w has a profound effect on the behavior of ordinal voting. The value of 0.5 should be used unless and until extensive and meaningful data is collected (a challenging task) which indicates a better value.

As w is decreased below 0.5, choices after the first have less and less influence over the outcome. When decreased all the way to $w = 0$, they have no influence and only first choices count. At $w = 0$, behavior is the same as Plurality and the IRV elimination method.

On the other hand, if w is increased, choices after the first exert a larger and larger influence upon the outcome. If increased all the way to $w = 1$, all choices count equally. Voters then lose the ability to specify which choice is their first. This is Approval Voting, which is inferior from the start since it fails to collect the most important first choice information (or in order to indicate a first choice, the voter must sacrifice the ability to vote for more than one candidate, which is what Approval Voting is supposed to be all about).

Thus, MRCV might be thought of as the optimum point between Approval Voting on one side and Plurality/IRV on the other.

9. But What Of Condorcet Winners, Favorite Betrayal, Participation, Paradoxes, IIA, Etc.?

Many contributors have produced a huge body of work over the past 250 years. There have been some interesting results, guidelines, theorems and voting paradoxes. However, this colossal amount of time and effort has failed to produce a solid consensus recommending any specific voting method. Various factions advocate different methods. Meanwhile, too many people in the world still suffer with Plurality. Why?

It has been shown that, in the perfect mathematical world and under some circumstances, every possible voting method will yield decisions which are "unfair" by some quite reasonable criteria. Every possible method also is in some way and to some degree susceptible to manipulation by strategic voting. (See results by Arrow, Gibbard, Satterthwaite, etc.)   A certain "paralysis" prevails with debates continuing as to which methods suffer from which defects, how often, and which defects are worse than which other defects. There is, however, some concern regarding how important said defects may or may not be in real-life elections. In order to do the math, some assumptions must virtually always be made.

It is not possible in real-world elections to have a complete ranking of all options by all voters. Even if complete rankings could in some way be forced, the data would then have a lower signal-to-noise ratio. The real world is a messy place. As has been explained, the best data that can be obtained from voters is very ragged, consisting of just first choices from some voters, first and second choices from others and perhaps three choices from a few. Therefore, any mathematics based upon the assumption that a complete ranking is available cannot be applied to real-world voting methods because at least one of the assumptions it is based upon is not valid.

A second choice carries only half the weight of a first choice, and a third only half of a second choice. Interchanging a second and third choice quite properly has much less impact on deciding the winner than swapping a first and a second choice. Therefore, any mathematics based upon the assumption that all choices carry equal weight cannot be applied to real-world voting methods because at least one of the assumptions it is based upon is not valid.

Consider the concept of a "Condorcet winner." A voting method which does not always elect a Condorcet winner when one exists is widely distained and unceremoniously cast upon the rejects pile. The existence of a Condorcet winner is determined by pairwise comparisons where a voter ranking of A over B in the 3^{rd} and 4^{th} places (or even the 98^{th} and 99th places) carries the same A-versus-B weight as a voter ranking of A over B in the 1^{st} and 2^{nd} places! This is not reality.

Whenever an option receives a majority of first choices, it is decisively a Condorcet winner and clearly must be respected. But any reasonable voting method handles this simple case correctly, even Plurality. The challenge for voting methods is to also render the correct result when no option has received a majority of first choices.

Finally, it should be emphasized that "fairness" must not be the overriding criterion for a voting method. As stated in the introduction, the primary objective must be simply to render the best possible decision – that result which maximizes voter satisfaction. One would not expect these two desirable characteristics be strongly at odds with each other. However, if or when such a conflict may arise, it must be resolved in favor of maximizing voter satisfaction, not "fairness."

Certainly, after 250 years, a different approach is called for. Sincerely thank the mathematicians; they've had their chance. Physicists and engineers must now devise a straightforward solution which optimizes the real-world data collection and data processing mechanism of voting. A solution is needed which can be put into service as soon as possible to replace Plurality (and IRV wherever that method has been implemented).

It is argued that MRCV is that best possible ordinal voting method because it collects the most valid and useful data from voters, then does the best job possible of utilizing the data to yield the decision which maximizes voter satisfaction. That's what is important. No claim is made that MRCV is perfect. The point is that whatever foibles, defects or weaknesses MRCV may have, they are minimized to the greatest degree possible. In order to refute the claim that MRCV is the best possible ordinal method, it is necessary to specifically define some other ordinal method which can be shown to gather better data and/or process the data in such a way that consistently produces decisions which result in higher voter satisfaction than MRCV.

Of course, if MRCV truly is the best real-world voting method, one should expect it to also work well in the theoretical mathematical world. And it does behave well. Appendix A which follows revisits a collection of voting problems and "paradoxes" to compare the performance of MRCV with other methods. The reader is urged to try MRCV in any kind of election, real or contrived. It always renders reasonable choices and does not produce an awful result under any circumstances.

Appendix A – Illustrative Examples

This appendix demonstrates the behavior of MRCV by presenting a series of examples. Several of the examples have been obtained from the extensive prior work which has been published in this area and a few are new. Some have been used to illustrate how various voting methods can produce differing results under the same circumstances. Some have been cited as problematic or labeled "voting paradoxes." Most of these examples have been discussed for decades; some for centuries. It should be pointed out that there really is no such thing as a voting "paradox." There are merely hypothetical sets of voter preferences for which some voting methods render results which are fairly obviously incorrect.

The notation 5 A > B > C is used to indicate that 5 voters preferred option A to option B and option B to option C; or that they rank the options A over B over C. It is assumed that the reader is familiar with the Plurality method, the Pairwise Comparison method of Condorcet (with the associated concepts of a "Condorcet winner" and a "Condorcet loser"), the scoring method of Borda and the Plurality with Runoff method known as Instant Runoff Voting or IRV.

For convenience, all references are listed after this appendix as all of them pertain to the appendix examples, except for the first. Being a tabula rasa new effort to define and engineer the best possible ordinal voting method for use in real-world elections, the main body has only one reference: Reference 1, the 2004 Dasgupta and Maskin article which inspired and triggered the effort.

Example 1

This example was lifted from reference 2 where it was presented as the simplest form of what is known as Condorcet's paradox.

1 A > B > C
1 B > C > A
1 C > A > B

The above is what is called a "loop" or "majority cycle" and shows that, while each voter's preference ordering makes sense (is and must be transitive), the collective result does not have to be transitive or make sense. Clearly, by itself this is one form of a tie and any reasonable method will render that as the result. There are many ways that one voter's input can cancel out or offset another voter's input. Majority cycles are unlikely to occur as a real election's overall result, but they certainly can occur with subgroups of voters. That leads into the next, more interesting example.

Example 2

Also from reference 2, this example was originally discussed by Condorcet.

30 A > B > C
1 A > C > B
29 B > A > C
10 B > C > A
10 C > A > B

1 C > B > A

The above 81 voters have defined a Condorcet winner. It is A. However, a Borda count would insist that B is the winner. A Condorcet winner is considered sacrosanct by most. However, Donald Saari (reference 3) offered a clever and semi-convincing argument as to why B really deserves to be the winner. Notice that the above 81 voters can be separated into three groups as below.

10 A > B > C	1 A > C > B	20 A > B > C
10 B > C > A	1 C > B > A	28 B > A > C
10 C > A > B	1 B > A > C	

The left group of 30 voters forms a majority cycle and so do the 3 voters in the center group. According to Saari, the 33 voters in these two groups cancel each other out (Condorcet would agree) and should be disregarded. The decision, therefore, is made by the remaining 48 voters (of the right group) in favor of B, 28 to 20, (28 is a majority of the 48 voters, but not a majority of all 81 voters).

While clever, this view is not fundamentally good. Imagine telling voters that, if they happen to fall into a majority cycle (which few would even comprehend), their votes will be disregarded! It turns out that this view is not even correct. There is some useful information within those majority cycles that must not be ignored. Here is how MRCV works for this case. The original voter rankings are on the left and the MRCV weighted points determination is on the right (multiply the number of voters times the positional weighting factor under each option and sum the products to the bottom of each column).

			A	B	C
30	A > B > C	30	4	2	1
1	A > C > B	1	4	1	2
29	B > A > C	29	2	4	1
10	B > C > A	10	1	4	2
10	C > A > B	10	2	1	4
1	C > B > A	1	1	2	4
		MRCV Points ➔	213	229	125

The weakest candidate is clearly identified as C with 125 points, so C must be eliminated.

30 A > B
1 A > B
29 B > A
10 B > A
10 A > B
1 B > A

After eliminating C and promoting any lower choices, A is now clearly revealed to be the winner with 41 first choices which is a majority of the 81 voters. Especially take note that 10 of the voters in the left majority cycle had C as their first choice as did one of the voters in the middle majority cycle. Promoting their second choices (after C was eliminated) gave 1 more first choice to B and 10 more first choices to A, thus giving A the majority.

Example 3

The 17 voters below are voting for 3 candidates. The preferences are designed to be close in several ways so as to present a challenging situation. Note that voting methods struggle to choose a winner.

6 A > B > C
5 C > A > B
4 B > C > A
2 B > A > C

Pairwise Comparison calls it a three-way tie (there is no Condorcet winner).
Plurality yields a tie between A and B with 6 votes each.
Borda elects A with 19 points.
IRV elects A with 11 first choices (after eliminating C).
The MRCV handling is shown below.

			A	B	C
6	A > B > C	6	4	2	1
5	C > A > B	5	2	1	4
4	B > C > A	4	1	4	2
2	B > A > C	2	2	4	1
		MRCV Points ➔	42	41	36

MRCV's computed proxy for the original cardinal satisfaction data reveals that the three candidates do indeed have similar amounts of support. However, C's support is the lowest (36 points) so C will be eliminated.

6 A > B
5 A > B
4 B > A
2 B > A

With a solid majority of 11 first choices, A is shown to be the winner. But there is more. Reference 2 employed this case as a monotonicty test. Suppose the group of 2 voters swaps their first and second choices, moving A up to first place (now the same ordering as the group of 6 voters). One would expect that this would improve A's chances of winning, certainly not hurt it. The preferences now are:

6 A > B > C
5 C > A > B
4 B > C > A
2 A > B > C

Pairwise Comparison still calls it a three-way tie (there is no Condorcet winner).
Plurality now elects A with a plurality of 8 votes.
Borda still elects A, now with 21 points.
IRV completely flips and elects C (a majority of 9 first choices after now eliminating B instead of C)!
MRCV handling is shown below.

			A	B	C
6	A > B > C	6	4	2	1
5	C > A > B	5	2	1	4
4	B > C > A	4	1	4	2
2	A > B > C	2	4	2	1
		MRCV Points ➔	46	37	36

As expected, the reconstructed cardinal data proxies show that A's support increased, B's support decreased and C's support remains unchanged. C is still the weakest and is still eliminated, electing A with a majority of 11 first choices as before.

Example 4

A variation of the above monotonicity test is the so-called "no show paradox" of Fishburn and Brams (reference 4) as discussed in reference 2. Suppose 1608 voters have the following preferences.

417 A > B > C
82 A > C > B
143 B > A > C
357 B > C > A
285 C > A > B
324 C > B > A

Pairwise comparison elects B (the Condorcet winner).
Plurality elects C with 609 first choices (not a majority).
Borda elects B with 1741 points.
IRV elects B with a 917 majority of first choices after eliminating A.
MRCV elects B with a 917 majority of first choices after eliminating A.

However, as the story goes, there were two voters whose preferences were A > B > C who failed to show up at the polls. Had they shown, the first group of 417 voters would have been 419.

419 A > B > C
82 A > C > B
143 B > A > C
357 B > C > A
285 C > A > B
324 C > B > A

Pairwise comparison still elects B (still the Condorcet winner).
Plurality still elects C with the same 609 votes.
Borda still picks B, now with 1743 points.
IRV, however, flips and eliminates B, causing C to win with a majority of 966 first choices.
MRCV still eliminates A and elects B with a 917 majority.
So, if the voting method were IRV, the two voters were better off staying home as their second choice won. If they had voted, instead of helping their first choice, they would have caused their last choice to win.

Example 5

This is the "Multiple Districts Paradox" of Fishburn and Brams as presented in reference 2. The idea is that, when voters in separate precincts all elect the same candidate, the combined result for all precincts should be expected to be that same candidate. Below are Precincts #1 and #2, and the combined preferences. (The careful observer will note that this is the previous example split into two precincts.)

#1		#2		Combined	
160	A > B > C	257	A > B > C	417	A > B > C
0	A > C > B	82	A > C > B	82	A > C > B
143	B > A > C	0	B > A > C	143	B > A > C
0	B > C > A	357	B > C > A	357	B > C > A
0	C > A > B	285	C > A > B	285	C > A > B
285	C > B > A	39	C > B > A	324	C > B > A

Pairwise Comparison elects B in all three cases (the Condorcet winner for all three).
Plurality elects C in #1, B In #2 and C in the Combined precincts.
Borda elects B in #1, C in #2 and B in the Combined precincts.
IRV elects A in both #1, and #2, but then elects B in the Combined precincts!
MRCV elects B for all three voter populations.

Example 6

This example is provided as a test for the conviction of those who claim a Condorcet winner should always be elected whenever one exists. Consider first the simple case of the 2001 voters below.

1001	A > C > B
1000	B > C > A

By a single vote, A wins a close election with a 1001 vote absolute majority. Of course, A also is a Condorcet winner. But suppose two additional voters chime in with a different preference.

1001	A > C > B
1000	B > C > A
2	C > A > B

Suddenly, A no longer has a majority of first choices and is no longer a Condorcet winner. In fact, there is a new Condorcet winner in town and it is C. Should C win this election with 0.1% of the first choices? Realize that the same two voters can wreak the same dramatic change even if the 1001 voters were 100,000,001 and the 1000 voters were 100,000,000 instead; the two voters would then be all of 0.000001% of the first choices. Furthermore, suppose that there are six candidates and the two voters don't even like C very much and rank three candidates ahead of C.

1001	A > C > B > D > E > F
1000	B > C > A > F > E > D
2	D > E > F > C > A > B

The two voters have once again dethroned A and crowned C the Condorcet winner. And again, this is true even if we add any number of additional voters equally to the first two groups. Perhaps in our mathematically perfect imaginary world there may indeed be some circumstances where the Condorcet winner should not automatically be declared the winner of an election. If so, that begs the questions: What are the boundaries? How many votes should C need in order to win the election? An answer is provided by MRCV. Here is the result with three candidates.

			A	B	C
1001	A > C > B	1001	4	1	2
1000	B > C > A	1000	1	4	2
2	C > A > B	2	2	1	4
		MRCV Points ➔	5008	5003	4010

According to the proxies for total voter satisfaction, C is noticeably weaker than A or B and is eliminated. The second choice of the two voters becomes their first choice, thus giving a 3-vote margin of victory to A. If the two voters had preferred B as their second choice, B would have won with a 1-vote margin. Having the second choice of the two voters who liked C decide which of two candidates wins (when both were already within a vote or two of an absolute majority) seems much more reasonable than having a microscopic number of first choices win the election for C over two extremely strong candidates. In order to avoid being eliminated, C would need to garner more than one seventh of all the first choice votes. If C had received at least 334 first choices, then B would be eliminated and C would win handily with a majority of 1334 first choices. Is this "correct" and reasonable? The best estimate it is possible to construct of relative voter satisfaction for each candidate says that it is. (See also discussion in Appendix B.)

<u>Example 7</u>

It is now time to leave the fun (but imaginary) world where we pretend we have full, complete, sincere and meaningful ranking data from every voter and consider what a real-world election might look like. The voters who actually vote in a given race, obviously have all specified their first choice. However, not all will have made a second choice, and even fewer a third. Suppose there are four candidates on the ballot which are at least somewhat well-known. We will assume that any write-ins or other recipients of only a very few votes have already been eliminated. Here is what the remaining data set might look like. Note that there are 1000 voters, so it is easy to see the percentages.

		D	R	L	G
100	D	4	0	0	0
115	D > G	4	0	0	2
100	D > L	4	0	2	0
95	D > G > L	4	0	1	2
90	R	0	4	0	0
140	R > L	0	4	2	0
110	R > L > G	0	4	2	1
60	R > L > D	1	4	2	0
20	L	0	0	4	0

130	L > R	0	2	4	0
10	L > G	0	0	4	2
5	L > D	2	0	4	0
5	G	0	0	0	4
10	G > D	2	0	0	4
5	G > L	0	0	2	4
5	G > D > L	2	0	1	4
	MRCV Points ➔	1740	1860	1590	650

Pairwise Comparison elects no one (there is no Condorcet winner).
Plurality elects D with a 410-vote plurality (D 410, R 400, L 165, G 20).
Borda elects R with 930 points.
IRV elects R (after first eliminating G and then L).
MRCV shows G to be decisively weakest, but the other three surprisingly close. Eliminate G.

		D	R	L
100	D	4	0	0
115	D	4	0	0
100	D > L	4	0	2
95	D > L	4	0	2
90	R	0	4	0
140	R > L	0	4	2
110	R > L	0	4	2
60	R > L > D	1	4	2
20	L	0	0	4
130	L > R	0	2	4
10	L	0	0	4
5	L > D	2	0	4
5		0	0	0
10	D	4	0	0
5	L	0	0	4
5	D > L	4	0	2
	MRCV Points ➔	1770	1860	1700

Note that D picked up 15 first choices after eliminating G and promoting lower choices, but does not achieve a majority. Also note that the five voters who had G as their first choice, but did not specify a second or third passed up any opportunity to further influence the outcome. R did not pick up any additional support. L picked up 5 first choices and 1010 more points, but not quite enough to avoid elimination in this round.

		D	R
100	D	4	0
115	D	4	0
100	D	4	0

95	D	4	0
90	R	0	4
140	R	0	4
110	R	0	4
60	R > D	2	4
20		0	0
130	R	0	4
10		0	0
5	D	4	0
5		0	0
10	D	4	0
5		0	0
5	D	4	0
	MRCV Points ➔	1840	2120

Now R has picked up 130 additional first choices, enough to win with a majority of 530 first choices. Perhaps this is not terribly surprising as the MRCV points indicated from the start that R had the most support. But, all-in-all, it was a pretty exciting election.

Two highly interesting observations: Except for cases where a candidate garners a majority of first choices, a Condorcet winner looks like a very rare occurrence in real-life elections. Furthermore, none of the defects that have absorbed so much time, attention and concern for years seems very likely to occur in actual elections either!

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Appendix B – Voting Method Considerations

Here, some overall requirements for good voting methods are listed. Comments are then offered for each of the methods most often discussed.

Minimum Requirements for Acceptable Voting Methods

1. It must be possible to have a durable audit trail to support transparent verification from each voter's input through to the final results (not often a problem, but needs to be mentioned).
2. The (Douglas) Jones rule must be satisfied: Everything about an election must be understandable by a bright high school student.
3. Voters must understand what information they are to provide, be able to provide it and be willing to provide it. Voters must not be forced to provide any information they are not already motivated to willingly provide.
4. Voters must be motivated to provide honest and sincere information; not insincere or strategic inputs. There must not be any way understandable by a substantial fraction (>5%) of voters to manipulate the results by providing insincere strategic information.
5. The voter's first choice is the most important single datum, so voters must have a way to indicate which option is their first choice.
6. Just the voter's first choice is insufficient information to make good decisions. Each voter must be allowed to indicate at least a second choice (except, of course, for binary decisions).
7. An option which receives an absolute majority of first choices must be declared the winner (the majoritarian rule).
8. All information provided by the voter must be best utilized to determine which option is the winner and/or which option is to be eliminated.

Plurality

Requirements 4 and 6 are not satisfied. Plurality works well *only* when there are just two options. When there are more than two, amazingly powerful "vote-for-the-lesser-of-two-evils" pressure frequently motivates voters to vote strategically and insincerely. If no option has received a majority of first choices, plurality is totally incapable of rendering an intelligent decision. Plurality's problems are widely recognized and it is regarded as the worst possible voting method. Simplicity is its only advantage. That and inertia account for its continued widespread use. A growing number of experts think use of plurality is a significant cause contributing to political polarization.

IRV or Instant Runoff Voting or Plurality With Runoff

Requirement 8 is not met. IRV does allow voters a first and (at least) a second choice. It is an improvement that the second choice will count as the first whenever a voter's first choice is eliminated. The pressure to vote insincerely is relieved. However, only first choice information is used when making the crucial decision as to which option is to be eliminated. The option which should win can be eliminated almost as easily as any other option. Thus, IRV, like plurality, is totally incapable of rendering an intelligent decision when no option has received a majority of first choices. Several of the examples in Appendix A emphasize IRV's erratic behavior.

Pairwise Comparison

Requirement 2 is not satisfied. It is not hard to understand Condorcet's underlying concept, but its implementation may be a bit beyond the easy comprehension of most voters. A larger drawback is that it is an incomplete method. There are many cases where no Condorcet winner exists. Condorcet called these ties, but most are not irreconcilable ties and can/should be resolved to choose a winner. So, to make it a complete method, other procedures are often tacked on to more correctly resolve those cases which have no Condorcet winner. Then, requirement 2 is *really* violated. Finally, there is a set of cases where a Condorcet winner does exist, but credible arguments can be made that the Condorcet winner should not, in fact, win the election. (See Appendix A, Example 6.)

Borda

There are many closely related variants, all of which violate various combinations of requirements 3, 4, 7 & 8. Borda scoring tends to favor options which have broad support, so much so that an option having broad support may win over an option which has received a majority of first choices. Borda is more susceptible than most methods to strategic manipulation.

Approval Voting

Requirements 4, 5 & 6 are not met. There is no provision for a voter to indicate a first choice. By not collecting that most important single datum, AV starts with a fatal information disadvantage. Many voters will want to indicate their first choice and so will be motivated to vote insincerely by indicating just one option. Then, of course, the voter has given up the ability to provide other choices, which is what AV is supposed to be all about. Ordinal systems forego the rich information conveyed by cardinal data; AV sacrifices ordinal data as well; not much is left. AV is a strange duck; neither fish nor fowl; not cardinal, but not ordinal either; definitely not the way to make the best decisions.

Score Voting or Range Voting

Requirement 4 is not satisfied. Most voters will figure out reasonably quickly that they can exert maximum impact without penalty by voting either zero or the maximum score allowed. So, score voting will degenerate over time into Approval Voting as voters gravitate to voting strategically. Game over. Score Voting seems very exciting because the use of sincere cardinal data would indeed enable much better decisions to be made. But if meaningful and sincere cardinal data could actually be obtained, then it would be good to also allow voters to indicate dissatisfaction as well as satisfaction.

True Weight Voting or TWV

TWV is a new approach that was proposed in 2016 and would satisfy all requirements. It promises to actually gather sincere, meaningful cardinal data from voters – both satisfaction and dissatisfaction – and to utilize that data in the best possible way. Theoretically, it would be fantastic. However, it is a radical change which is completely untested and speculative at this time.

MRCV

MRCV meets all requirements. It is argued that MRCV is the best possible voting method when restricted to use of ordinal data. Whenever no option has a majority of first choices, the current ordinal data set is used to generate a proxy for voters' total cardinal satisfaction for each option. The cardinal data proxies are used to determine and iteratively eliminate the weakest option until one option has a majority of first choices (or is the only remaining option in rare cases).

MRCV aims exclusively at rendering the best decision, defined to be the option that results in the greatest voter satisfaction totaled over all those who voted, period. It is possible to construct sets of voter preferences, primarily ones which will not occur in real elections, for which MRCV will render a result to which some may object. For example, MRCV almost always elects a Condorcet winner (when one exists), but will not elect a Condorcet winner in situations where the Condorcet winner would not result in the highest voter satisfaction (see Appendix A, Example 6).

The behavior of MRCV is affected by the choice of the weighting factor it uses. As previously explained, a weighting factor of 1/2 is employed as the best assumption that can be made and MRCV behavior is quite good. Interestingly, if the weighting factor were increased, say to 2/3, the set of hypothetical cases (complete and meaningful rankings from all voters, etc.) for which MRCV may not render a result consistent with a fairness axiom is greatly reduced. Whether or not this would be a good thing is highly debatable since such hypothetical cases seldom, if ever, occur in real elections. The primary objective is to make the best decisions, not "fairness." (But what could be fairer than always choosing the option that maximizes voter satisfaction?)

It would be interesting and perhaps useful to measure the weighting factor for many races and many elections in order to determine an overall mean value and standard deviation. That would be an interesting but difficult project which would require lots of data for many races and many elections. Unless and until such valid data is available which indicates that a different weighting factor better matches reality, the value of 1/2 should be used.